Top

Left side

TEST

Right side

Bottom

the lectures pdfs are available at:

https://www.physics.umd.edu/rgroups/amo/orozco/results/2022/Results22.htm

Correlations in Optics and Quantum Optics; A series of lectures about correlations and coherence 1. November 2022

Luis A. Orozco
www.jqi.umd.edu
BOS.QT

Lesson 3

Tentative list of topics to cover:

- From statistics and linear algebra to power spectral densities
- Historical perspectives and examples in many areas of physics
- Correlation functions in classical optics (field-field; intensity-intensity; field-intensity)
- Correlation functions in quantum examples
- Correlations and conditional dynamics for control
- Correlations in quantum optics of the field and intensity
- Optical Cavity QED
- From Cavity QED to waveguide QED.

- M. Born and E. Wolf, *Principles of Optics* Cambridge University Press, Cambridge,1999, 7th expanded.

- L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* Cambridge University Press, Cambridge, 1995.

- E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light* Cambridge University Press, Cambridge, 2007.

Wave equation of a scalar field *V*(**r**,*t*)

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)V(\mathbf{r},t) = 0,$$

$$U(\mathbf{r},\omega) = \int_{-\infty}^{\infty} V(\mathbf{r},t)e^{i\omega t}\mathrm{d}t,$$

$$(\nabla^2 + k^2)U(\mathbf{r},\omega) = 0,$$

Normalized mutual coherence function:

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau) = \langle V^*(\mathbf{r}_1,t)V(\mathbf{r}_2,t+\tau)\rangle. \quad \gamma(\mathbf{r}_1,\mathbf{r}_2;\tau) = \frac{\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)}{\sqrt{I(\mathbf{r}_1)I(\mathbf{r}_2)}},$$

Intensity is the mutual coherence at equal time t and position r

$$I(\mathbf{r}) = \Gamma(\mathbf{r},\mathbf{r};0) = \langle V^*(\mathbf{r},t)V(\mathbf{r},t)\rangle$$

The cross-spectral density is the Fourier transform of the correlation (normalized as the spectral degree of coherence).

$$W(\mathbf{r}_1,\mathbf{r}_2;\omega)=\int_{-\infty}^{\infty}\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)e^{i\omega\tau}\mathrm{d}\tau.$$

$$W(\mathbf{r}_1,\mathbf{r}_2;\omega)=\langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle,$$

Spectral degree of coherence and its bounds:

$$\mu(\mathbf{r}_1,\mathbf{r}_2;\omega)=\frac{W(\mathbf{r}_1,\mathbf{r}_2;\omega)}{\sqrt{S(\mathbf{r}_1,\omega)S(\mathbf{r}_2,\omega)}} \qquad 0\leq|\mu(\mathbf{r}_1,\mathbf{r}_2;\omega)|\leq 1$$

Spectral density (intensity at frequency ω)

$$S(\mathbf{r},\omega)=W(\mathbf{r},\mathbf{r};\omega)$$

Modern coherence theory began in 1954 when Wolf found that the function of mutual coherence in free space satisfies the wave equations:

$$\left(\nabla_1^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Gamma(\mathbf{r}_1, \mathbf{r}_2; t) = 0,$$

$$\left(\nabla_2^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Gamma(\mathbf{r}_1, \mathbf{r}_2; t) = 0,$$

The cross spectral density (Fourier transform of the correlation) also satisfies the Helmhotz equation:

$$(\nabla_1^2 + k^2)W(\mathbf{r}_1,\mathbf{r}_2;\omega) = 0,$$

$$(\nabla_2^2 + k^2)W(\mathbf{r}_1,\mathbf{r}_2;\omega) = 0,$$

With its associated diffraction integrals.

Knowing the cross spectral density W(0) in the plane of the object allows in principle the calculation of the cross spectral density anywhere on z>0.

Quantum regression theorem

- Correlation functions can be calculated using a master equation with the appropriate initial and boundary conditions (Lax 1968).
- This recalls the propagation of the correlations using the wave equation for the electromagnetic (Wolf 1954, 1955)

For vector beams we take the electric cross spectral density matrix, which can be used to characterize the coherence state and the polarization state in the source plane z=0 is (ρ_i is a point in the source plane):

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = \begin{pmatrix} W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) & W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) \\ W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) & W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) \end{pmatrix},$$

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = \langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle, \quad (i,j=x,y).$$

Then at the observation point for two points r_i E is linked to the source by a Green function or a diffraction integral

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2;\omega) = \langle E_i^*(\mathbf{r}_1,\omega)E_j(\mathbf{r}_2,\omega)\rangle, \quad (i,j=x,y).$$

Since we are actually talking about vector fields, the spectral degree of polarization of the field at any point is defined as the ratio of the intensity of the polarized part of the beam to the total intensity:

$$\mathcal{P}(\mathbf{r},\omega) = \sqrt{1 - \frac{4 \det \mathbf{W}(\mathbf{r},\mathbf{r};\omega)}{[\mathrm{Tr}\,\mathbf{W}(\mathbf{r},\mathbf{r};\omega)]^2}},$$

With field spectral density (with a prefactor) :

$$S(\mathbf{r},\omega) = \mathrm{Tr}\,\mathbf{W}(\mathbf{r},\mathbf{r};\omega).$$

The degree of polarization is bounded from non-polarized (0) to fully polarized (1):

$$0 \leq \mathcal{P}(\mathbf{r},\omega) \leq 1$$

The Stokes spectral parameters are combinations of the elements of the electric cross spectral density matrix:

$$s_0(\mathbf{r},\omega) = W_{xx}(\mathbf{r},\mathbf{r};\omega) + W_{yy}(\mathbf{r},\mathbf{r};\omega),$$

$$s_1(\mathbf{r},\omega) = W_{xx}(\mathbf{r},\mathbf{r};\omega) - W_{yy}(\mathbf{r},\mathbf{r};\omega),$$

$$s_2(\mathbf{r},\omega) = W_{xy}(\mathbf{r},\mathbf{r};\omega) + W_{yx}(\mathbf{r},\mathbf{r};\omega),$$

$$s_3(\mathbf{r},\omega) = i[W_{yx}(\mathbf{r},\mathbf{r};\omega) - W_{xy}(\mathbf{r},\mathbf{r};\omega)].$$

The degree of polarization is related to the correlation function and this changes depending on the position as the wave propagates.

When talking about correlation functions, Wolf's and Glauber's notation are different.

Wolf uses Greek letters ($\gamma^{(4)}$) with the index associated with the number of fields.

Glauber uses Latin letters ($g^{(2)}$) with the index associated with the number of intensities.

Tiny are normalized functions; uppercase letters are not normalized.

The polarization coherence theorem, a recent colorary:

$$P^2 = V^2 + D^2, \text{ and } V^2 + D^2 \leq 1$$

The degree of polarization squared P is the sum of the squares of visibility V and distinguishability D; all associated with the elements of the electric cross spectral density matrix.

J. H. Eberly, X.-F. Qian, and A. N. Vamivakas, “Polarization coherence theorem,” Optica **4**, 1113 (2017).

A. F. Abouraddy, “What is the maximum attainable visibility by a partially coherent electromagnetic field in Young’s double-slit interference?” Opt. Express **15**, 18320 (2017).

Measurements of Correlations in Optics

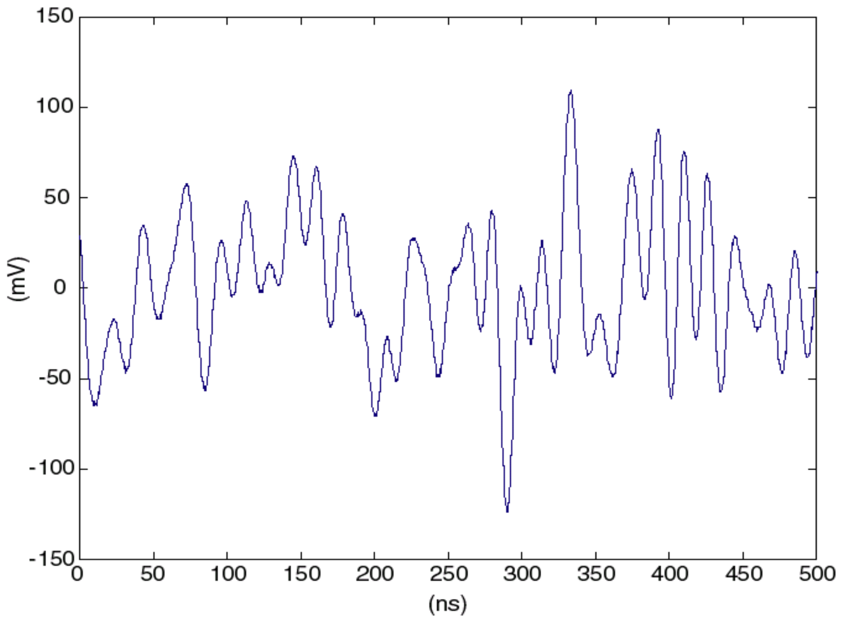
The study of noisy signals uses correlation functions.

$\langle F(t)\, F(t+\tau) \rangle$

$\langle F(t)\, G(t+\tau) \rangle$

For optical signals the variables usually are Field and Intensity.

Photocurrent with noise:



How do we measure these functions?

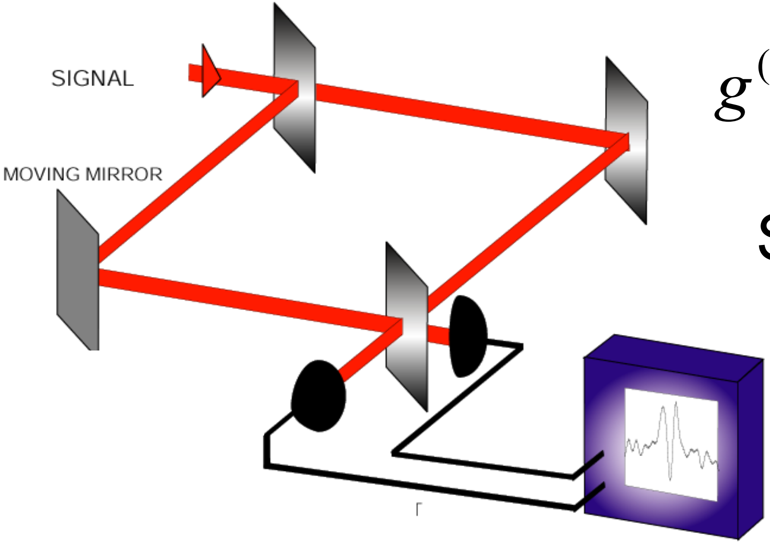
$G^{(1)}(\tau) = \langle E(t)^* E(t+\tau) \rangle$ field-field

$G^{(2)}(\tau) = \langle I(t) I(t+\tau) \rangle$ intensity-intensity

$H(\tau) = \langle I(t) E(t+\tau) \rangle$ intensity-field

- Correlation functions tell us something about fluctuations.
- The correlation functions have classical limits.
- They are related with conditional measurements. They give the probability of an event given that something has happened.

Mach Zehnder or Michelson Interferometer Field –Field Correlation



$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle I(t)\rangle}$$

Spectrum:

$$F(\omega) = \frac{1}{2\pi}\int exp(i\omega\tau)g^{(1)}(\tau)d\tau$$

This is the basis of Fourier Spectroscopy

Michelson interferometer

Interference of two waves

$$E_1 e^{i\phi_1} + E_2 e^{i\phi_2}$$

$$I = |E_1 e^{i\phi_1} + E_2 e^{i\phi_2}|^2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2)$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

for equal phases

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Fringe visibility

in general:

$$V = \frac{4\sqrt{I_1 I_2}\left\langle \cos(\phi_1 - \phi_2) \right\rangle}{2(I_1 + I_2)}$$

for equal intensities:

$$V = \left\langle \cos(\phi_1 - \phi_2) \right\rangle$$

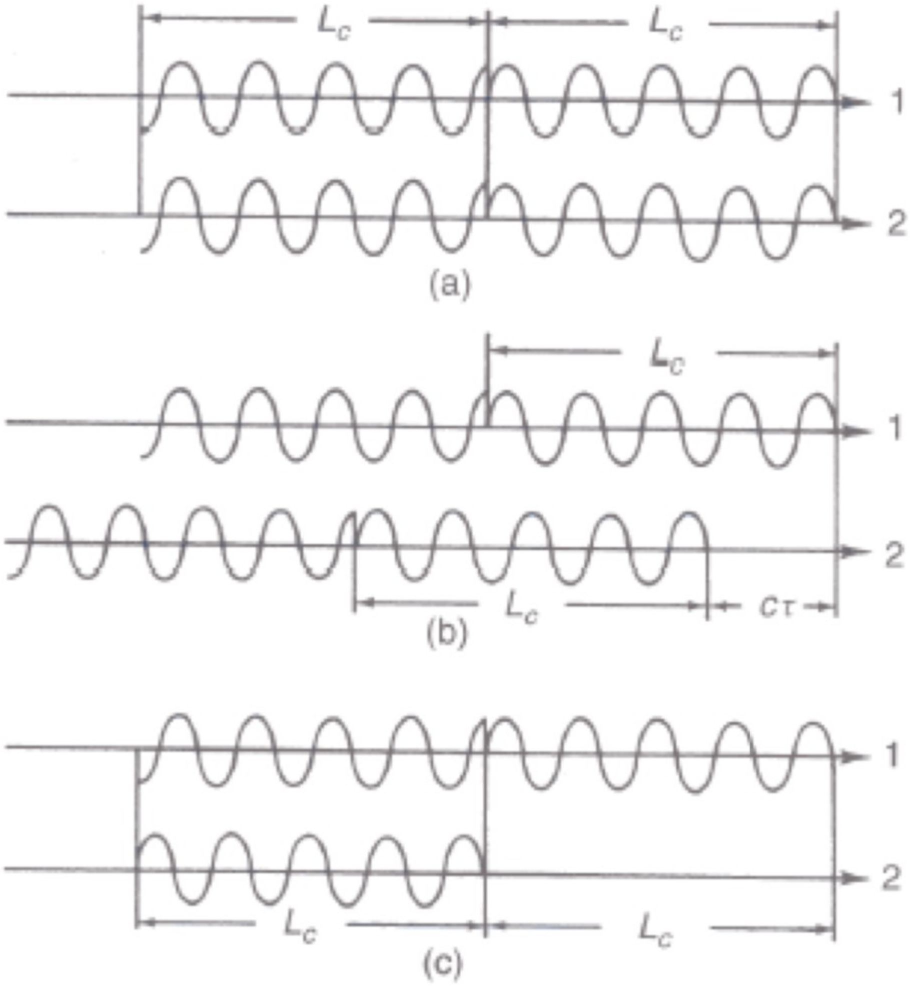
Visibility measures the ability to interfere, the coherence. You can see it looks like equal time correlation function $<E^*_1E_2>$

If the phases are random the average of cosine is zero.

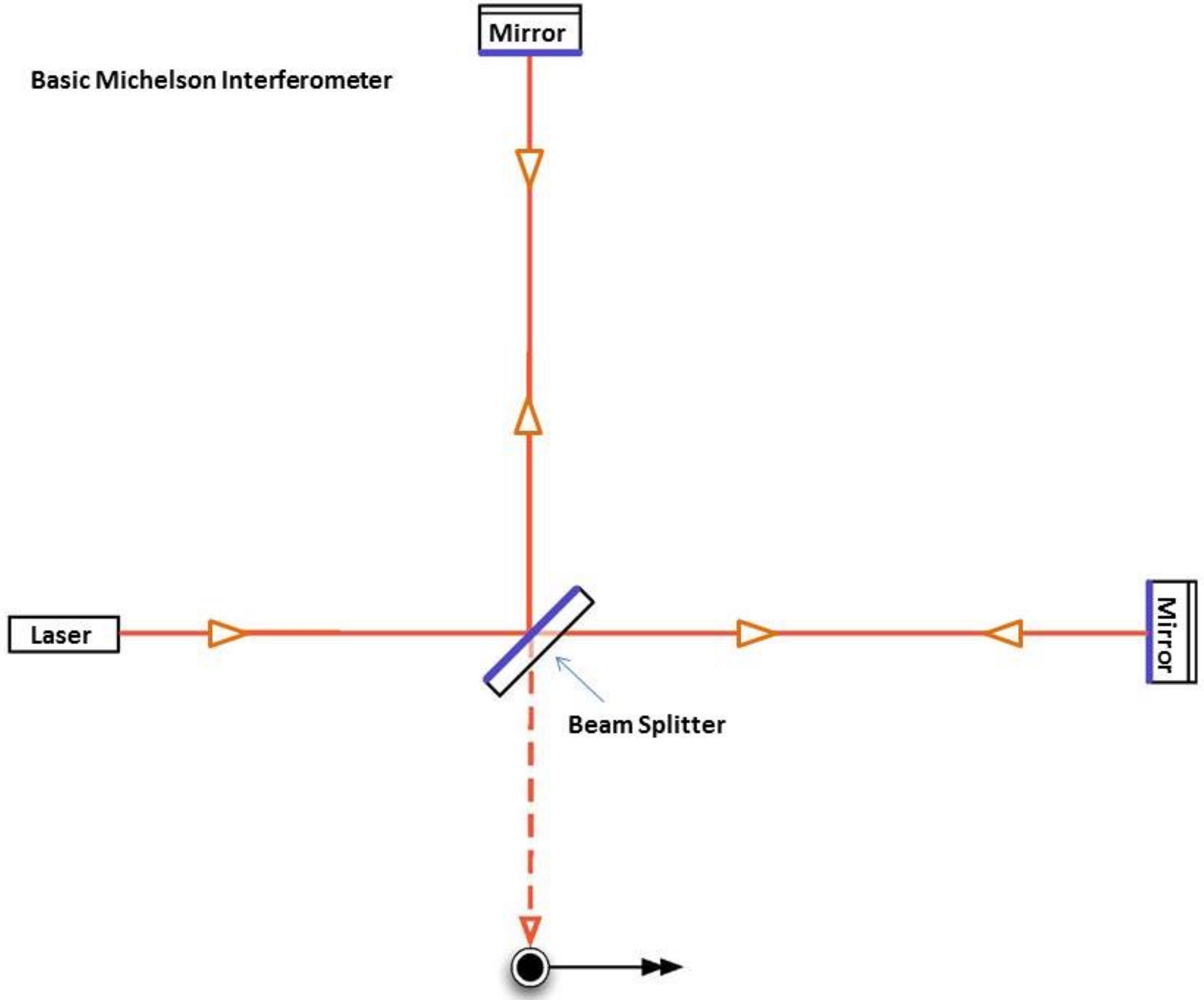
Visibility is zero.
If they have a distribution that average will be between ± 1 and the visibility will be between 1 and 0

The interference term (cosine) is related to coherence, is a correlation.

- How do we measure that ability to interfere?
- Michelson interferometer, as we can vary the phase difference
- What can affect interference?
- The inequality of amplitudes
- The wavefronts are not equal
- Time frequency and wavenumber
- Polarization
- Interruptions in the phase, phase jumps



L_c
L_c
1
2
(a)
L_c
1
2
L_c
$c\tau$
(b)
1
2
L_c
L_c
(c)


Basic Michelson Interferometer
Mirror
Laser
Beam Splitter
Mirror

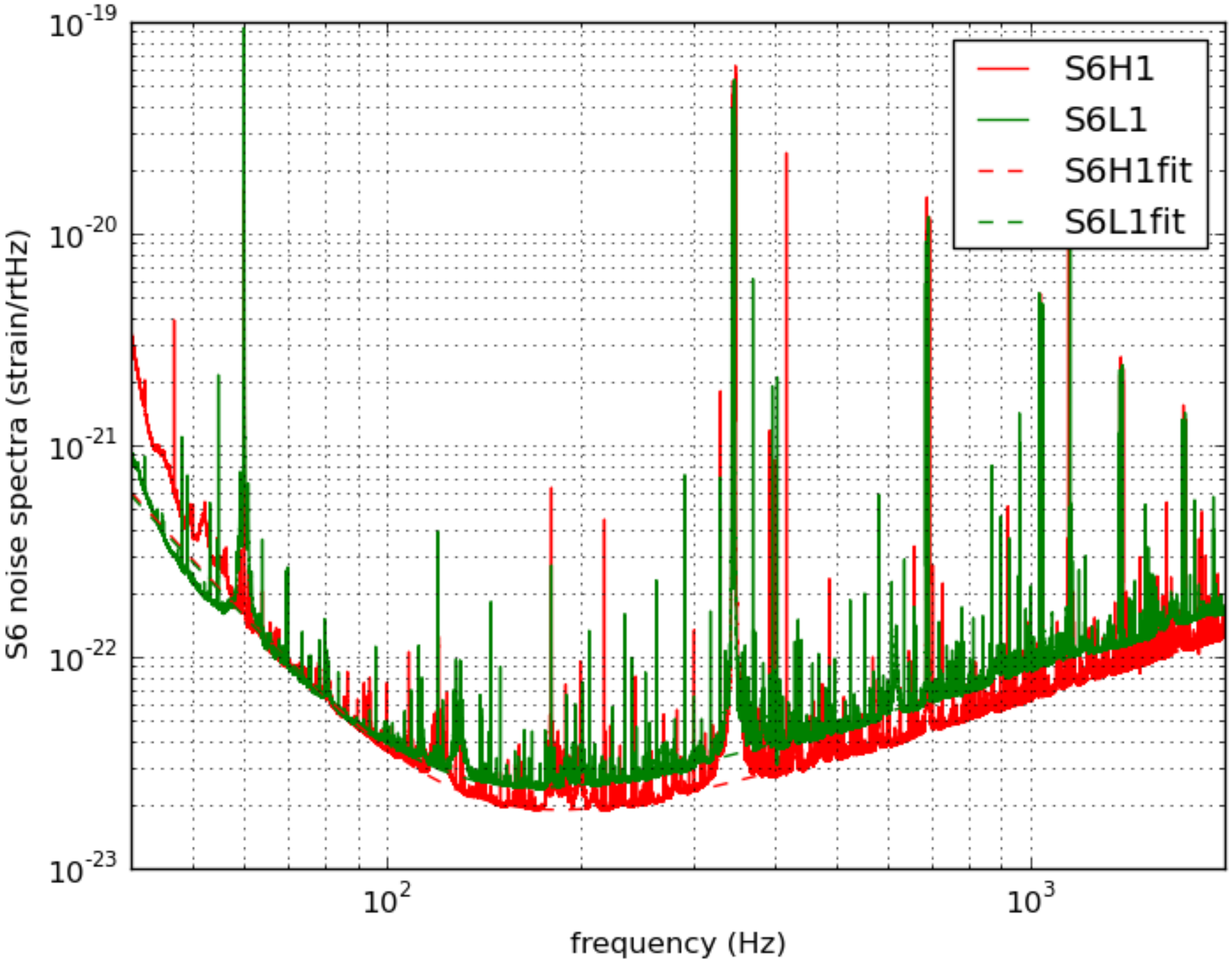
Remember that in a beam splitter (semitransparent mirror) there is a difference in the phases depending on whether the reflection is from higher to lower or if it is from lower to higher refractive index. T-invariance (Stokes argument).

LIGO and VIRGO

The LIGO and VIRGO signal is a small change Δd in the separation in the interferometer arm of length d causing a change in phase $\delta \pm \varepsilon$ that changes as the wave passes.

Measure $h=\Delta d/d$
The signal is the difference $S=I(\delta+\varepsilon)-I(\delta+\varepsilon)$

What is your current noise limit?



S6H1
S6L1
S6H1fit
S6L1fit
S6 noise spectra (strain/rtHz)
frequency (Hz)
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{2}
10^{3}

LIGO has 4 km in each arm and VIRGO 3 Km

$$h = 2 \times 10^{-23} = \Delta d/d$$

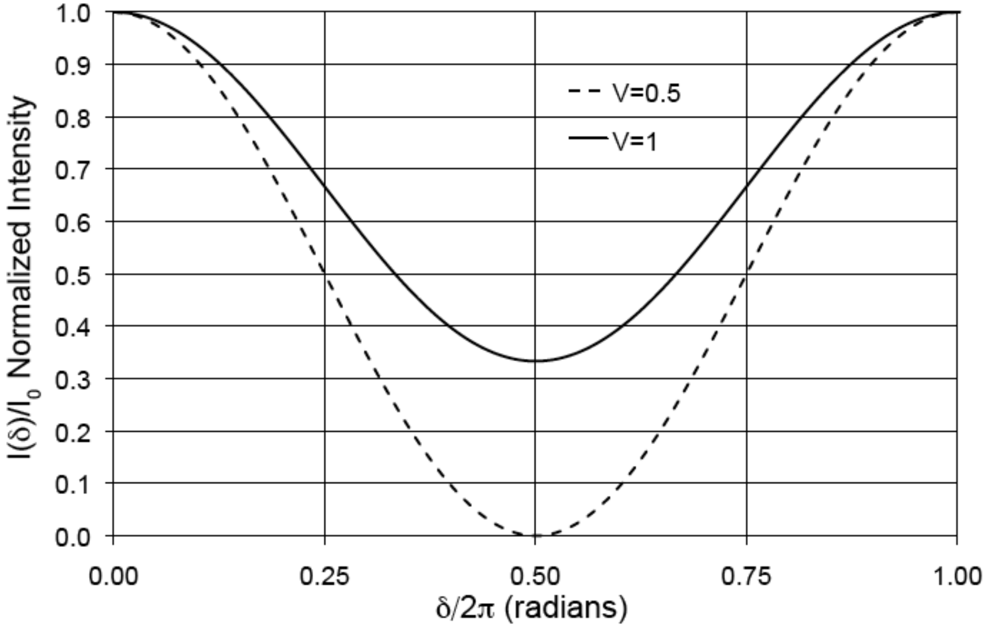
$$\Delta d = 2 \times 10^{-23} \times 4 \times 10^{3}$$

$$\Delta d = 8 \times 10^{-20}\ \text{m}$$

The radius of a proton is 1×10^{-15} m

If d is the distance between the earth and the sun
The sensitivity Δd is one-tenth of the Bohr radius.

Where to operate the LIGO and VIRGO interferometer, in the linear part or at the ends?



The signal is the difference due to displacement

$$I_{out} = I_0 \left(A + B \cos^2 \frac{\delta}{2} \right)$$

$$\begin{aligned}
S &= I_0 \left(\cos^2 \frac{\delta - \epsilon}{2} - \cos^2 \frac{\delta + \epsilon}{2} \right) \\
S &= I_0 \left(\left(\cos^2 \frac{\delta}{2} + \epsilon \sin \frac{\delta}{2} \cos \frac{\delta}{2} \right) - \left(\cos^2 \frac{\delta}{2} - \epsilon \sin \frac{\delta}{2} \cos \frac{\delta}{2} \right) \right) \\
S &= 2 I_0 \epsilon \sin \frac{\delta}{2} \cos \frac{\delta}{2}
\end{aligned}$$

The maximum signal is in the linear part in the phase

$$
\begin{aligned}
\frac{\partial S}{\partial \delta} &= I_0 \epsilon (\cos^2 \delta/2 - \sin^2 \delta/2) \\
\frac{\partial S}{\partial \delta} &= 0 \\
\delta/2 &= \pi/4
\end{aligned}
$$

But the important thing is not the maximum signal but the maximum signal in the signal-to-noise ratio:

$$\frac{S}{N} = \frac{I_0 \cos^2 \frac{\delta+\epsilon}{2} - I_0 \cos^2 \frac{\delta+\epsilon}{2}}{\sqrt{I_0 \cos^2 \frac{\delta+\epsilon}{2} + I_0 \cos^2 \frac{\delta+\epsilon}{2}}}$$

$$N \approx \sqrt{2 I_0 \cos^2 \frac{\delta}{2}} = \sqrt{2 I_0} \cos \frac{\delta}{2}$$

$$\frac{S}{N} = \frac{2I_0\epsilon\sin\frac{\delta}{2}\cos\frac{\delta}{2}}{\sqrt{2I_0}\cos\frac{\delta}{2}}$$

$$= \sqrt{2I_0}\epsilon\sin\frac{\delta}{2}$$

$$\frac{\partial}{\partial\delta}\left(\frac{S}{N}\right) = \frac{\sqrt{2I_0}}{2}\epsilon\cos\frac{\delta}{2}$$

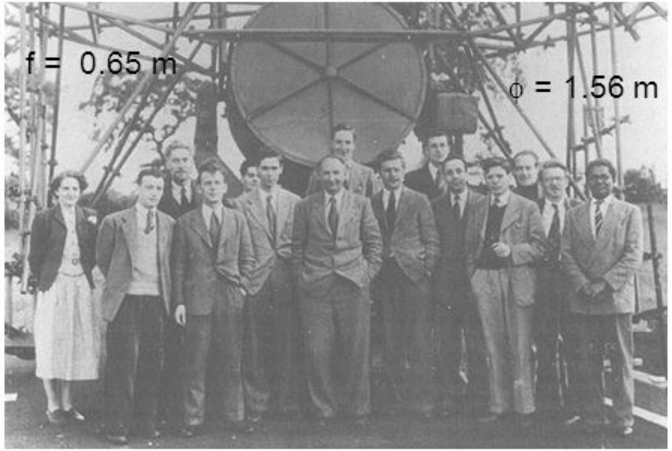
At the minimum part is a maximum, the S/N is linear in e

If they are in the dark it is easier to identify an increase in intensity.
In the linear part, there is already a lot of light, shot noise.

In reality LIGO and VIRGO operate near that point, but they take into account other noises in their measurement.

The Hanbury Brown Twiss stellar interferometer, Mk 1

R. Hanbury Brown & R. Q. Twiss,
A Test of a New Type of Stellar Interferometer on Sirius ,
Nature **178**, 1046-1053 (1956).



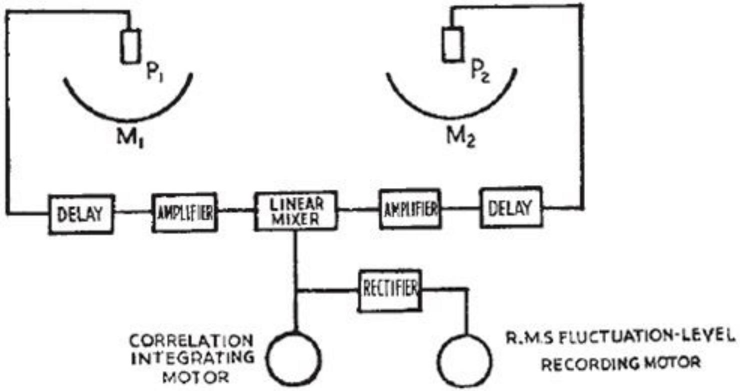


Fig. 1. Simplified diagram of the apparatus

Prediction from astrophysical theory is 0.0063"

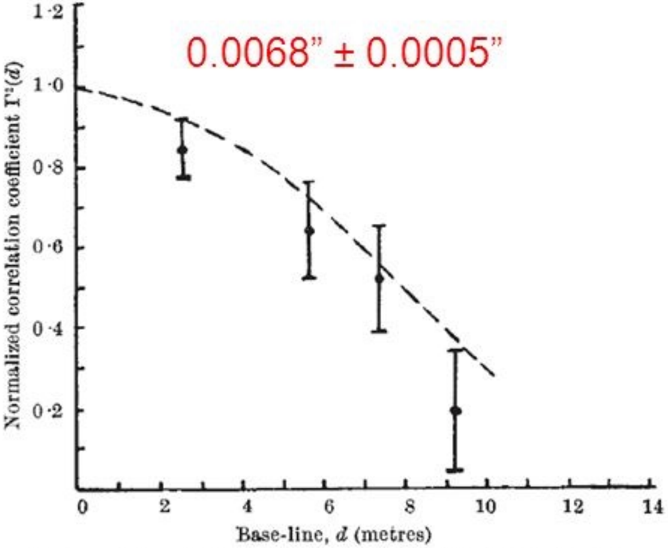
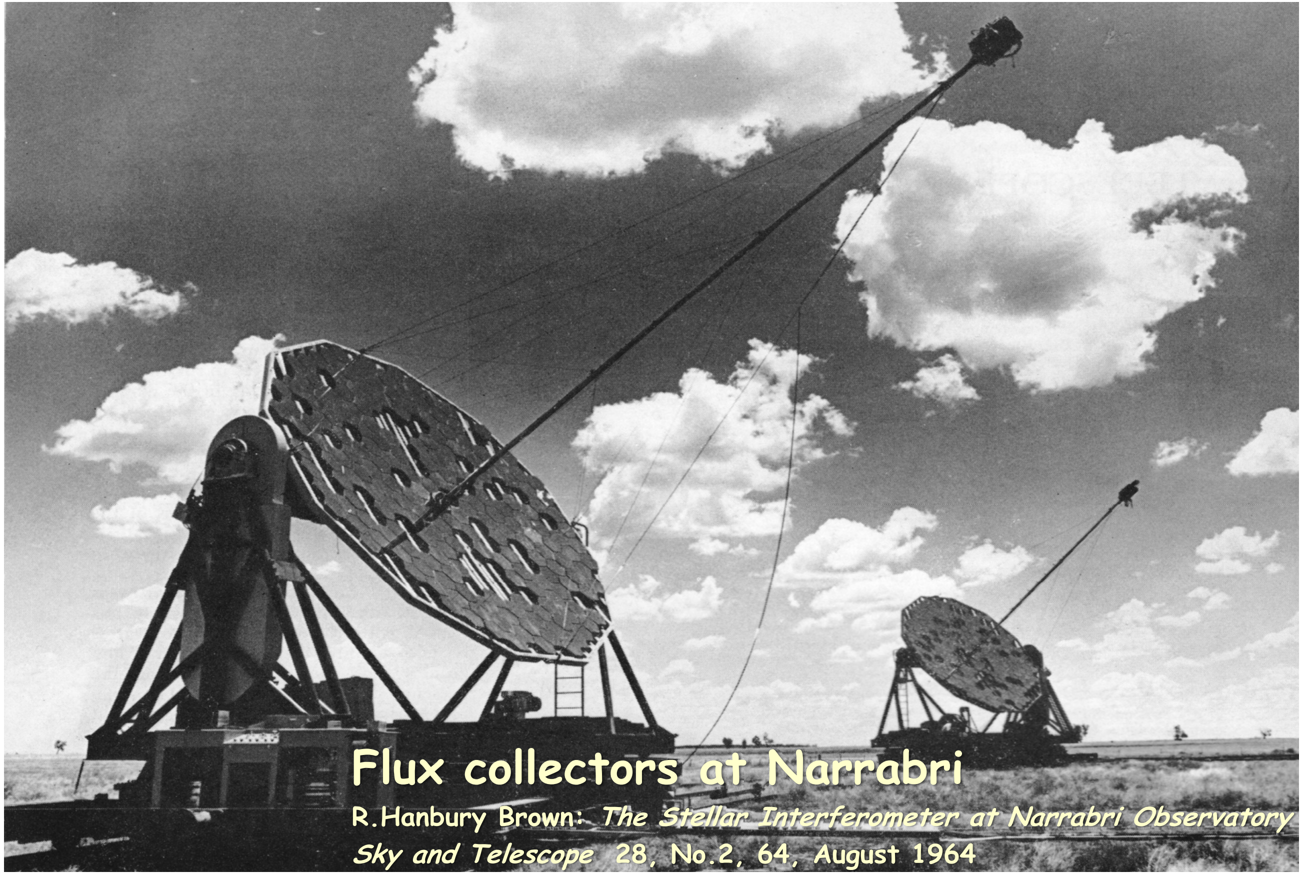
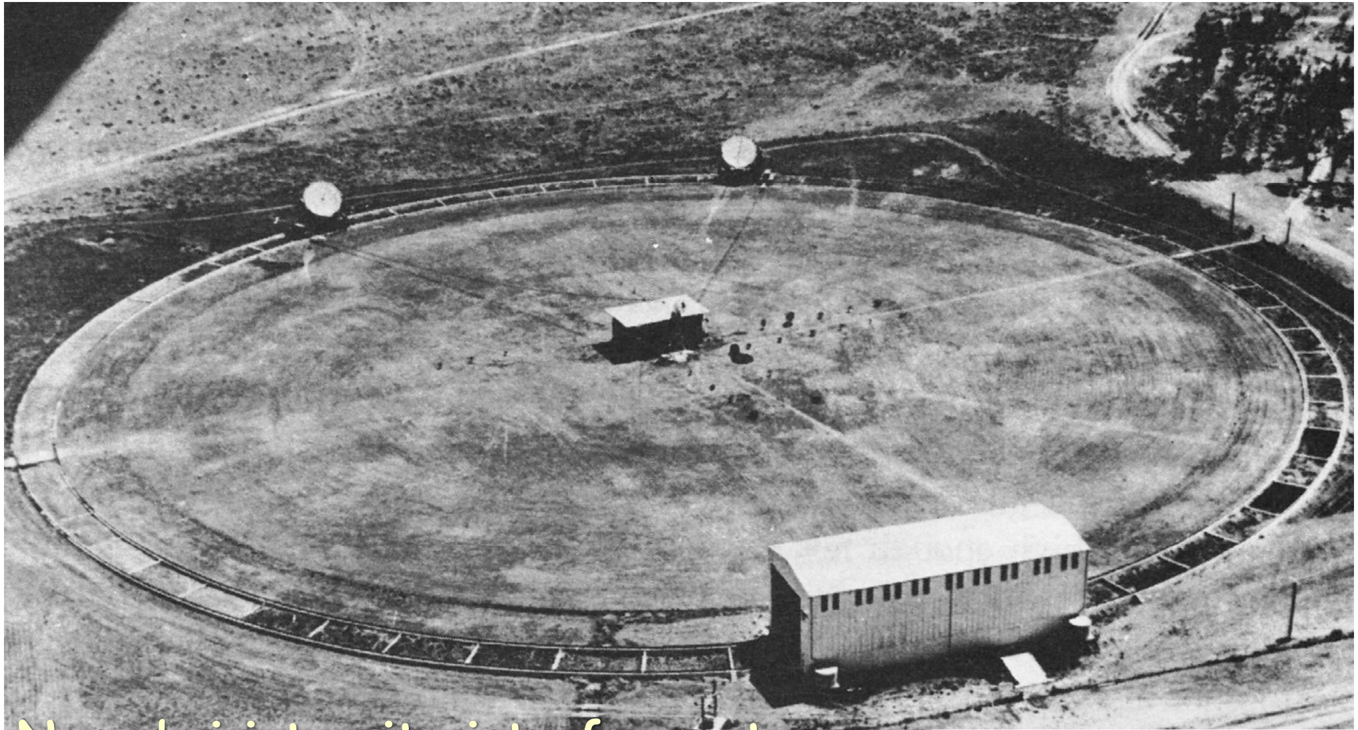


Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0·0063". The errors shown are the probable errors of the observations

First measurement of stellar diameter in 30 years

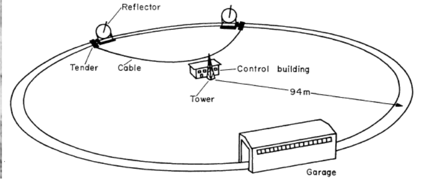

Flux collectors at Narrabri
R.Hanbury Brown: *The Stellar Interferometer at Narrabri Observatory*
Sky and Telescope 28, No.2, 64, August 1964



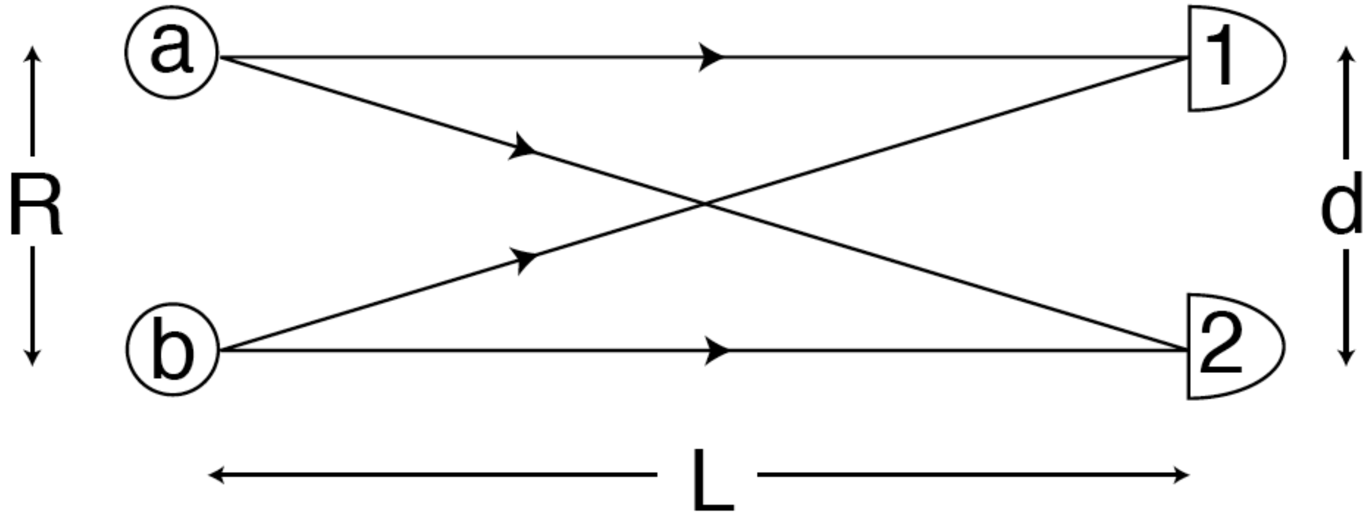
Narrabri intensity interferometer

with its circular railway track

R.Hanbury Brown: *BOFFIN. A Personal Story of the Early Days of Radar, Radio Astronomy and Quantum Optics* (1991)



The HBT controversy



The physics of Hanbury Brown–Twiss intensity interferometry: from stars to nuclear collisions.*

GORDON BAYM

Presented at the XXXVII Cracow School of Theoretical Physics, Zakopane, Poland.
May 30 - June 10, 1997.

Source a and b are within a Star. Can we measure the angular distance R/L~θ so that we could know the diameter?

Source a:

$$\alpha e^{ik|\vec{r}-\vec{r}_a|+i\phi_a}/|\vec{r}-\vec{r}_a|$$

Source b:

$$\beta e^{ik|\vec{r}-\vec{r}_b|+i\phi_b}/|\vec{r}-\vec{r}_b|$$

The amplitude at detector 1 from sources a and b is:

$$A_1 = \frac{1}{L}\left(\alpha e^{ikr_{1a}+i\phi_a} + \beta e^{ikr_{1b}+i\phi_b}\right)$$

And the intensity I_1

$$= \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2 + \alpha^*\beta e^{i(k(r_{1b}-r_{1a})+\phi_b-\phi_a)} + \alpha\beta^* e^{-i(k(r_{1b}-r_{1a})+\phi_b-\phi_a)}\right)$$

The average over the random phases ϕ_a and ϕ_b gives zero

$$\langle I_1\rangle = \langle I_2\rangle = \frac{1}{L^2}\left(\langle|\alpha|^2\rangle + \langle|\beta|^2\rangle\right)$$

And the product of the intensity of each of the detectors $\langle I_1\rangle\langle I_2\rangle$ is independent of the separation of the detectors.

Multiply the two intensities and then average.

$$\langle I_1 I_2 \rangle = \langle I_1 \rangle \langle I_2 \rangle + \frac{2}{L^4} |\alpha|^2 |\beta|^2 \cos\left(k(r_{1a} - r_{2a} - r_{1b} + r_{2b})\right)$$

$$= \frac{1}{L^4} \left[(|\alpha|^4 + |\beta|^4) + 2|\alpha|^2|\beta|^2 (1 + \cos\left(k(r_{1a} - r_{2a} - r_{1b} + r_{2b})\right)) \right].$$

$$g^{(2)} = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}$$

$$= 1 + 2 \frac{\langle |\alpha|^2 \rangle \langle |\beta|^2 \rangle}{(\langle |\alpha|^2 \rangle + \langle |\beta|^2 \rangle)^2} \cos\left(k(r_{1a} - r_{2a} - r_{1b} + r_{2b})\right).$$

$$(L \gg R),\ k(r_{1a} - r_{2a} - r_{1b} + r_{2b}) \rightarrow k(\vec{r}_a - \vec{r}_b) \cdot (\hat{r}_2 - \hat{r}_1) = \vec{R} \cdot (\vec{k}_2 - \vec{k}_1)$$

This function changes as a function of the separation between the detectors.

$d = \lambda/\theta$ with $\theta = R/L$

Relation to the Michelson Interferometer

$$|A_1 + A_2|^2 = |A_1|^2 + |A_1|^2 + (A_1^* A_2 + A_1 A_2^*)$$

The term in parenthesis is the associated to the fringe Visibility (first order coherence) if we now take the square of the fringe visibility and average it:

$$\langle V^2 \rangle = 2\langle |A_1|^2 |A_2|^2 \rangle + \langle A_1^{*2} A_2^2 \rangle + \langle A_1^2 A_2^{*2} \rangle$$

$$\langle V^2 \rangle \rightarrow 2\langle I_1 I_2 \rangle$$

The solution of E. M. Purcell, *Nature* **178**, 1449 (1956).

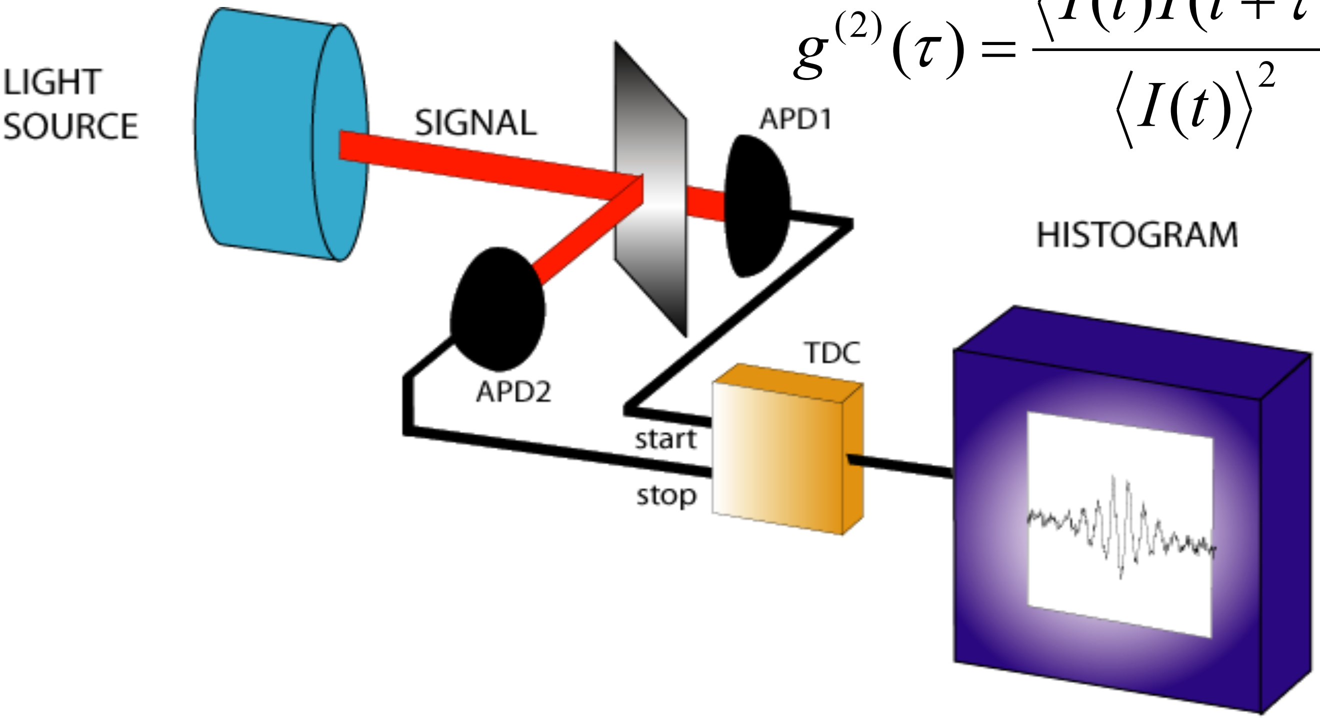
Mentions the work of Forrester first real optical intensity correlation. A. T. Forrester, R. A. Gudmundsen and P. O. Johnson, "*Photoelectric Mixing of Incoherent Light,*" Phys. Rev. **99**, 1691 (1955).

Mentions that bosons tend to appear together

Does the calculation and relates it to the first order coherence.

Hanbury Brown and Twiss; Intensity Intensity correlation

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$



R. Hanbury Brown and R.Q. Twiss, Correlation between Photons in Two Coherent Beams of Light, Nature 177, 27 (1956).

Correlations of the intensity at τ=0

$$g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2}$$

$$= \frac{\langle (I_0 + \delta(t))^2 \rangle}{\langle I_0 + \delta(t) \rangle^2}$$

$$= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

It is proportional to the variance

Thanks